requires that the sum of the squares of the errors of the coordinates, or variables measured, shall be a minimum. And since the law of errors in all observations is the same, which law is given by the equation

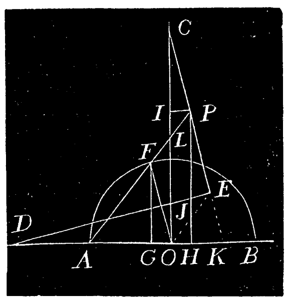
$$y = \frac{p}{1 + cx^2},$$

(ANALYST, p. 51 Vol. V), where y is the probability of the error of magnitude x; therefore, universally, when the constants in an equation or equations, of any number of variables, are to be determined from measured values of the variables, they must be determined by the condition that the sum of the squares of the errors of these measured values shall be a minimum.

DEMONSTRATION OF THE CISSOID.

BY JAMES SIMMONS, JR., BELOIT, WIS.

THE following construction is founded on the definition of the Cissoid given in Olney's General Geometry and Calculus. CO is the fixed line, D the fixed point, CED the right angle, whose side $CE = DO$. Required to find the locus of its middle point P.



With a center O, and radius CP describe the semicircle. Join O and E, O and F, A and P Draw PH and FG parallel to CO; PI to BD, and produce CE to K.

Since CEJ and DOJ are equal right angles $JE = JO$, and the triangle OJE is isosceles; $\therefore$ $JEO = JOE$; $\therefore$ $PEO = DOE$; $\therefore$ $OEK = EOK$, and the triangle OKE is isosceles.

But $PE = AO$; $\therefore$ triangle AKP is isosceles, AP is parallel to OE, and $EPO = OAP$. Now since $OFA = OAF$, FO is parallel to PE and $OFG = PCI$. But since $FO = CP$, CIP and FGO are equal right triangle and $IP = GO$, and the point P describes the cissoid according to the definition.

To obtain the equation of the cissoid, make $CE = AB = 2a$; $AH = x$; $PH = y$. Then, from the similar triangles AHP and AOL, we get

$$x : y :: a : OL.$$

But $OL = \frac{1}{2}CO = \frac{1}{2}[y + CI\,(= FG)] = \frac{1}{2}\{ y + \sqrt{[(2a - x)x]} \}$;

$$\therefore x : y :: a : \tfrac{1}{2}\{ y + \sqrt{[(2a - x)x]} \};$$

which gives $$y^2 = \frac{x^3}{2a - x},$$ the equation of the cissoid.